Name: _____

Section 13.10
Diversity of Biological Populations

Introduction This activity starts with reading the first few pages of a scientific article on the diversity of biological populations. We proceed to answering some valuable mathematical questions. As a conclusion, we use Lagrange multipliers to find an extreme value.

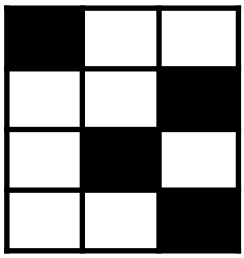
Article Source http://digitalassets.lib.berkeley.edu/math/ucb/text/math_s5_v4_article-12.pdf

Procedure

(1) Read the scientific article from p163 (the beginning) to the middle of p165 just below formula 2.4. As you read the text, assume that $\log$ is actually the natural logarithm $\ln(x)$.

(2) In the table on the back page, select one question per row to answer. Ignore the other two questions on that row. Additionally, you cannot select two successive questions in the same column.

OK



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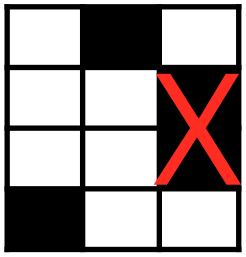

(3) As you answer questions, you may write them down or record them, whichever is more suitable.

(4) The last question (Lagrange multipliers) must be completed by all students. Therefore, each students will complete a total of five tasks.

(5) Each row is worth 10 points. The total is 50 points including the last question.

Try Sterling's approximation, namely $\ln(x!) \approx x\left(\ln(x)-1\right)$, for a few selected values of your choice. Don't attempt to prove it! Then use it to show the steps from formula 2.2 to formula 2.4.	Explain formula 2.1 (ignore the log). Give a few examples using letters of the alphabet.	Explain the meaning of N, N_i, s, B, H, H', p_i. Explain these values not by copy-pasting the text, but as if you were explaining to somebody *who had already read the article and didn't understand*.
Search online for data about two or three colleges you are interested in, regarding the student population. Compute H' to determine the numerical diversity of each college. Comment on the results, draw conclusions. Also, the article mentions exclusively animals and plants… do you think it is ok to use this formula for student populations?	Go outside, find a spot of land with some kind of plants or animals. Count the numbers of each species and compute the diversity. Your sample should have more at least 30 individuals. Take a picture of the place you selected. Attach your picture with some explanations. Also collect a few plants such that the bunch has a value of H' between 1.3 and 1.6.	Discuss whether a population composed of two individuals of species A and one individual of species B is more or less diverse than a population with twenty and ten individuals. Use both the formulas for B and for H' for this task. Pick more examples if you want. Comment on the results, draw conclusions.
Discuss the significance of choosing another base of the log compared to the natural log. In particular, when does it yield H' = 1 ? (Hint, for log in base n, study the case with n species).	Suppose there are *at most* five species of trees in a park for a total of 200 trees. Pick up some numbers and compute the diversity H' for each (formula 2.4). Make choices that will draw attention to some properties of formula 2.4. Comment on your observations.	In formula 2.4, let's consider the situation where there are *n* types of flowers and all of them are in equal proportion. Discuss $\lim_{n\to\infty} H'$.
Ask your own (high-level) mathematical question on the subject introduced in the article and attempt to answer it. It should be a theoretical question, not a question of simple computations.	Read any part(s) of the article beyond the assignment and discuss interesting informations that you understand. Write a paragraph.	The article lists three ways of assigning a number to diversity. One is to simply count the number of species. The other two are B (formula 2.1) and H' (formula 2.4) (or H). Come up with a fourth formula of your making that assigns a number to diversity. Apply it to a few examples. Explain its merits and weaknesses. (Obviously we are not expecting a ground-breaking discovery but a fair intellectual endeavor).

Given *n* species, what proportions yield the highest diversity according to H' ? Justify using Lagrange multipliers. Start with the case of two species with $p_1 = x$ and $p_2 = y$. Then try three species with $p_1 = x$, $p_2 = y$ and $p_3 = z$, etc. Then infer the general case for *n* species. (Use your TI-Nspire!)