Name: ________________

Divisibility — Bus stop Unjumble Activity

Reference: Exercises for chapter 4 (p100): exercises 6, 8.

Activity Here are three propositions.

(a) Suppose $a,b,c \in \mathbb{Z}$. If $a \mid b$ and $a \mid c$, then $a \mid (b+c)$.

(b) Suppose $a \in \mathbb{Z}$. If $5 \mid 2a$, then $5 \mid a$.

(c) Suppose $a \in \mathbb{Z}$. If $7 \mid a-1$, then $7 \mid a^2-1$.

i. Illustrate each proposition with two examples.

ii. Below, you will find the steps in the proof for the three propositions. They are all scrambled together. Put them back in order, first proposition (a), then (b), then (c). Some lines are missing that you will need to complete. Once you placed all the proofs back in the correct order, the letters will form a sentence. Find out what it is!

Scrambled steps of the three proofs

E. Then $a^2-1=(a+1)\cdot(a-1)=(a+1)\cdot 7x = 7\cdot\big(x(a+1)\big)=7\cdot n$ where $n = x(a+1) \in \mathbb{Z}$.

R. This means $x = 2n$ for some $n \in \mathbb{Z}$.

B. Then $b+c = ax+ay = a(x+y) = az$ where $z = x+y \in \mathbb{Z}$.

A. Since $a \mid b$ and $a \mid c$, then $b = ax$ for some $x \in \mathbb{Z}$ and $c = ay$ for some $y \in \mathbb{Z}$.

A. Since $5x = 2a$, we conclude that $5x$ is an even number, so x is even.

D. Because $a^2-1 = 7\cdot n$ for some $n \in \mathbb{Z}$, $7 \mid a^2-1$.

S. Since $5 \mid 2a$, then $2a = 5x$ for some $x \in \mathbb{Z}$.

I. We conclude that $a = 5n$ for some $n \in \mathbb{Z}$, so $5 \mid a$.

R. (Missing step)

U. (Missing step)

V. (Missing step)

Letters in order from proof (a) to proof (c)

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