

## Linear Algebra Trigonometric Identities

Concept: in this worksheet, we will prove two trigonometric identities that we studied in Precalculus.

Consider the two matrices  $M_\alpha = \begin{bmatrix} \cos \alpha & -\sin \alpha \\ \sin \alpha & \cos \alpha \end{bmatrix}$  and  $M_\beta = \begin{bmatrix} \cos \beta & -\sin \beta \\ \sin \beta & \cos \beta \end{bmatrix}$

**Question 1** Describe the effect of  $M_\alpha$

**Question 2** Describe the effect of  $M_\beta$

*There are two ways we could consider a rotation of angle  $\alpha + \beta$  ...*

- Either we can simply write a matrix  $M_{\alpha+\beta}$  following the same principle as the matrices written above (see question 3).*
- Or we can consider it to be a composition of the two matrices  $M_\alpha$  and  $M_\beta$ , by multiplying them (see question 4).*

**Question 3** Write the matrix  $M_{\alpha+\beta}$  that would produce a rotation of angle  $\alpha + \beta$

**Question 4** Calculate the product  $M_\alpha \cdot M_\beta$ .

*These two techniques should yield the same result.*

**Question 5** Conclude the proof of two trigonometric identities by comparing the matrices in questions 3 and 4.