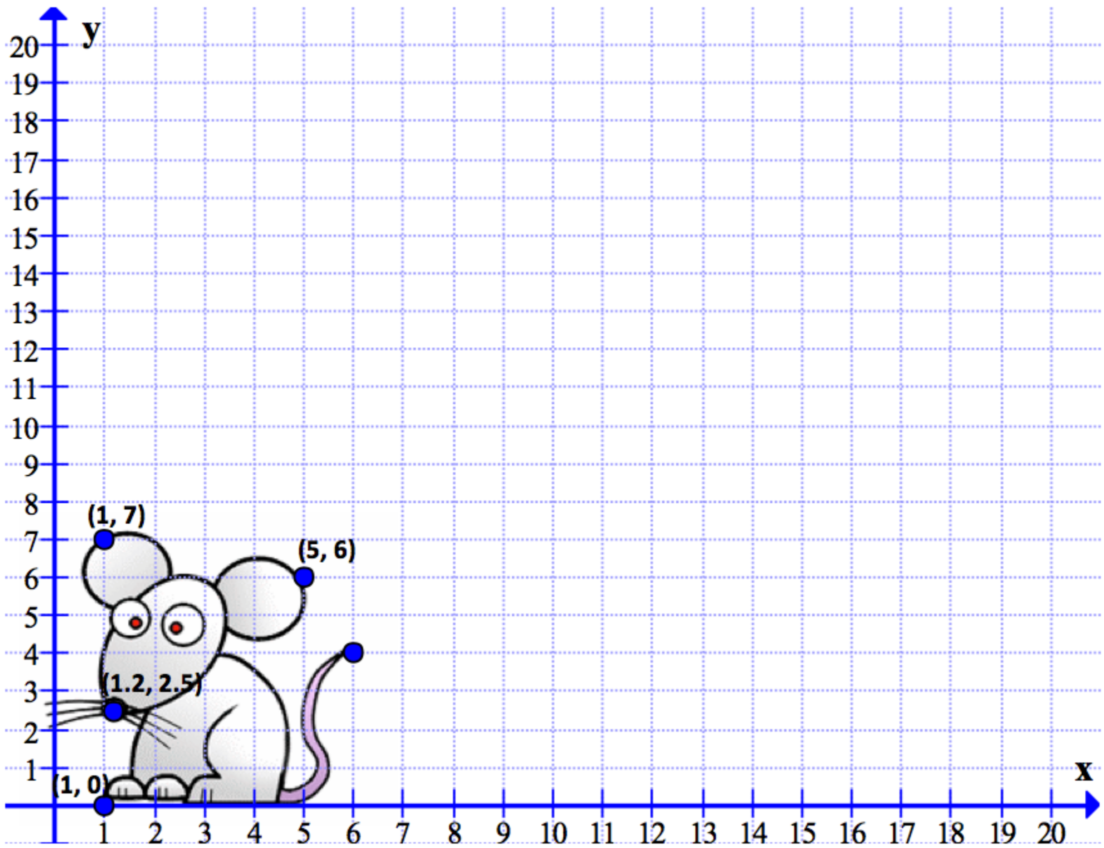
Name: ____________________

Linear Algebra
Rotating Matrices



Activity 1: Scaling matrix This warm-up is not a rotation! Given the five points shown on the drawing of the mouse, use matrix multiplication with the matrix $A = \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}$ to find the position of the new points. Place them and draw the new mouse on the same set of axes:

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \qquad \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} 1.2 \\ 2.5 \end{bmatrix} =$$

$$\begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} = \qquad \begin{bmatrix} 3 & 0 \\ 0 & 3 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} =$$

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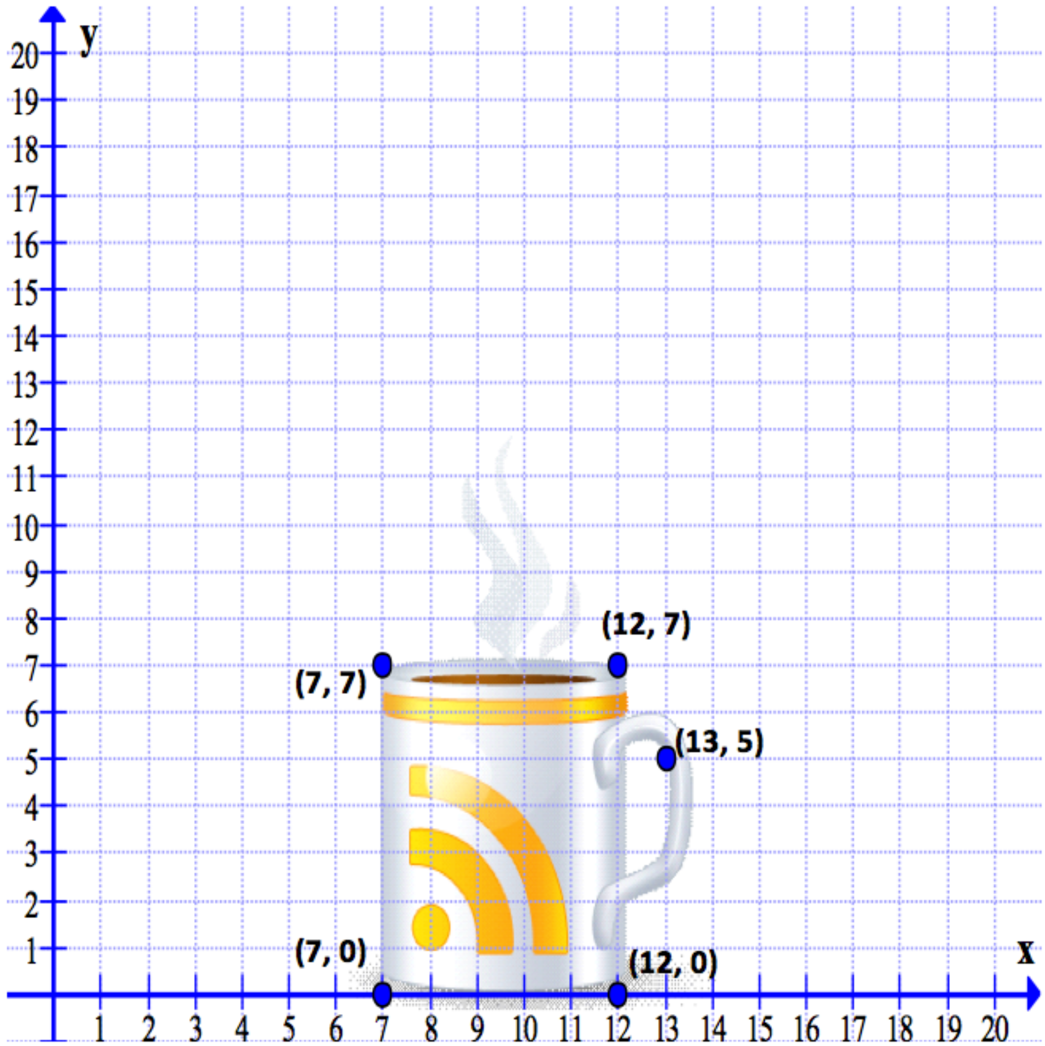


<u>Activity 2: Rotation matrix</u> Do the same with the cup of coffee, using the matrix

$B = \begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}$. What is the angle of rotation for matrix B?

$$\begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} = \qquad\qquad \begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} =$$

$$\begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} = \qquad\qquad \begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} =$$

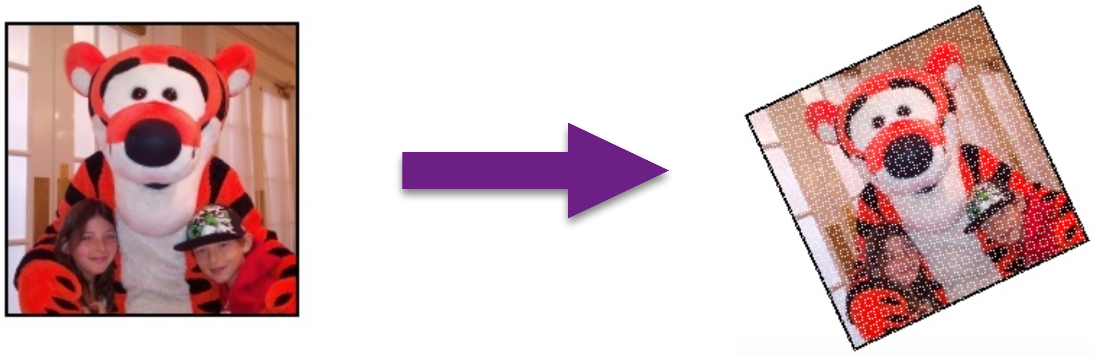
$$\begin{bmatrix} 0.64 & -0.77 \\ 0.77 & 0.64 \end{bmatrix}\begin{bmatrix} \quad \\ \quad \end{bmatrix} =$$

<u>Activity 3: Shear matrices to create rotation</u>

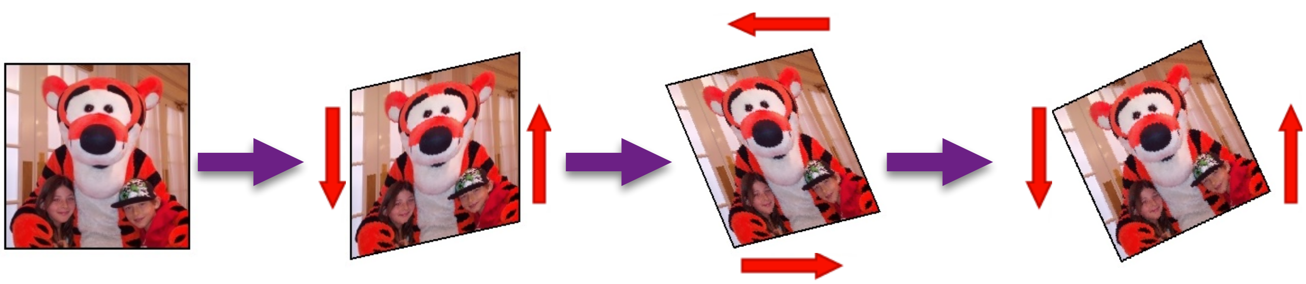
(Source: http://www.datagenetics.com/blog/august32013/index.html)

(Website to use shear (called "skew") on pictures: http://www192.lunapic.com/editor/?action=skew)

In reality, with computers, when we use rotation matrices to rotate pictures, the result doesn't come out perfectly, with white pixels appearing.



That is why a technique has been developed that uses three shears instead of one rotation! The result is better on a computer.



The three shears of the linear transformation are shown here:

$$T(\vec{v}) = \begin{bmatrix} 1 & -\tan(\alpha/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ \sin\alpha & 1 \end{bmatrix} \begin{bmatrix} 1 & -\tan(\alpha/2) \\ 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

<u>Exercise</u> Prove that the product of the three matrices above is equal to the matrix of rotation $B = \begin{bmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{bmatrix}$.