Name: ____________________

Linear Algebra
Binet's formula

In this in-depth discovery of some special features of the Fibonacci sequence, we will introduce, use and prove Binet's formula using linear algebra.

(a) Find "Binet's formula" on the internet and explain what it is used for.

(b) Find F_6, F_7, F_8, F_9 and F_{50}.

(c) Write the first 11 terms of the Fibonacci sequence, starting with

F_0	F_1	F_2	F_3	F_4
0	1	1		

(d) Consider the space *V* of all infinite sequences that satisfy $f_n = f_{n-1} + f_{n-2}$. The Fibonacci sequence is obviously an element of that space. Choose two other elements of that space and list the first five terms of each.

(e) What is the dimension of space V ?

(f) Write a basis of that space V.

(g) Find two elements in V that have the form: 1, r, r^2, r^3, … (Note: there in fact exist only two such elements)

(h) Is it possible to use these two elements as a basis of V? If so, write the Fibonacci sequence as a linear combination of these two sequences.

(i) Find F_i, the *i*-th term of the Fibonacci sequence, using your new basis.

(j) Research how the Fibonacci sequence appears in Pascal's triangle

(k) Even though the Fibonacci sequence as we defined it (0, 1, 1, 2, 3, 5…) has a first term F_0=0, we can work backwards and find F_{-1}, F_{-2}, etc. Do the formula $f_n = f_{n-1} + f_{n-2}$ and Binet's formula agree (check F_{-1}, F_{-2} and F_{-3})

(l) In the previous question, we extended the concept from natural numbers to integers. Can we try to extend the idea to all real numbers? Use Binet's formula with three non-integers F_x, F_{x+1} and F_{x+2}. Does the formula $f_n = f_{n-1} + f_{n-2}$ still hold? Comment on what you notice.

(m) Research online about graphs of the Fibonacci sequence for real numbers. Sketch the graph(s) that you find.

(n) Finally! Prove that the number of ways of putting n dominoes of size 2×1 in a $2 \times n$ box is the number F_{n+1}. (The illustration on the right shows 3 dominoes in a 2×3 box. The dots don't matter. There are $F_4 = 3$ ways to place them.)